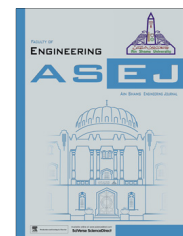




Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com



ENGINEERING PHYSICS AND MATHEMATICS

Effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet



Machireddy Gnaneswara Reddy ^{a,*}, Polarapu Padma ^b, Bandari Shankar ^c

^a Department of Mathematics, Acharya Nagarjuna University Campus, Ongole 523 001, India

^b Department of Mathematics, S. R. Govt. Arts & Science College, Kothagudem 507 101, India

^c Department of Mathematics, Osmania University, Hyderabad 500 007, India

Received 4 December 2014; revised 16 March 2015; accepted 10 April 2015

Available online 6 June 2015

KEYWORDS

Unsteady flow;
 Magneto hydrodynamics;
 Thermal radiation;
 Heat transfer;
 Stretching sheet;
 Viscous dissipation

Abstract The aim of this paper is to present the unsteady magnetohydrodynamic (MHD) boundary layer flow and heat transfer of a fluid over a stretching sheet in the presence of viscous dissipation and heat source. Utilizing a similarity variable, the governing nonlinear partial differential equations are first transformed into ordinary differential equations before they are solved numerically by applying Keller Box method. Effects of physical parameters on the dimensionless velocity and temperature profiles were depicted graphically and analyzed in detail. The numerical predictions have been compared with already published papers and good agreement is obtained. Finally, numerical values of physical quantities such as the skin friction coefficient and the local Nusselt number are presented in tabular form. Heat transfer rate at the surface increases with increasing values of Prandtl number and unsteadiness parameter whereas it decreases with magnetic parameter, radiation parameter, Eckert number and heat source parameter.

© 2015 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

The fluid flow over a stretching surface is important in applications such as extrusion, wire drawing, metal spinning, and hot rolling, etc [1–3]. It is crucial to understand the heat and

flow characteristics of the process so that the finished product meets the desired quality specifications. Since the pioneering work of Sakiadis [4,5], various aspects of the problem have been investigated by many authors. It should be noticed that there have been published several papers [6–15] on the flow and heat transfer problems for stretching surfaces.

The unsteady heat transfer problems over a stretching surface, which is stretched with a velocity that depends on time are considered by Anderson et al. [16], and a new similarity solution for the temperature field is devised, which transfers the time dependent thermal energy equation to an ordinary differential equation. Elbashbeshy and Bazid [17] studied the heat transfer over an unsteady stretching surface. Recently, Ishak et al. [18] have studied the heat transfer over an unsteady stretching vertical surface. Ishak et al. [19] have also

* Corresponding author.

E-mail addresses: mgrmaths@gmail.com (M. Gnaneswara Reddy), padmapolarapu@gmail.com (P. Padma), bandarishanker@yahoo.co.in (B. Shankar).

Peer review under responsibility of Ain Shams University.



Production and hosting by Elsevier

investigated the unsteady laminar boundary layer over a continuously stretching permeable surface, while Ali and Mehmood [20] have presented a study of homotopy analysis of unsteady boundary layer flow adjacent to permeable stretching surface in a porous medium. Very recently, Gnaneswara Reddy [21] presented an unsteady radiative convective boundary layer flow of a casson fluid with variable thermal conductivity.

Ishak [22] studied the MHD flow and heat transfer characteristics over an unsteady stretching surface. Yusof et al. [23] extended Ishak [22] work by introducing the effect of radiation for the MHD flow and heat transfer over an unsteady stretching surface. Radiation is energy that comes from a source and travels through some material or through space. Light, heat and sound are types of radiation. Radiation is considered in his study due to the fact that thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. Many new engineering processes such as fossil fuel combustion energy processes, solar power technology, astrophysical flows, gas turbines and the various propulsion devices for aircraft, missiles, satellites, and space vehicle re-entry occur at high temperature. So knowledge of radiation plays a very important role and hence, its effect cannot be neglected. Also thermal radiation is of major importance in many processes in engineering areas which occur at a high temperature for the design of many advance energy conversion systems and pertinent equipment. The Rosseland approximation is used to describe the radiative heat flux in the energy equation.

The objective of the present study is to extend the works of Yusof et al. [23] by introducing effects of viscous dissipation and heat source in the energy equation. The irreversible process by means of which the work done by a fluid on adjacent layers due to the action of shear forces is transformed into heat is defined as viscous dissipation. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flows such as injection molding or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed aircraft raises the temperature of the skin. Boundary layer flows with internal heat generation over a stretching sheet continue to receive attention because of its many practical applications in a broad spectrum of engineering systems.

2. Mathematical formulation

We consider the unsteady two dimensional laminar boundary layer flow past a continuously stretching sheet immersed in an incompressible electrically conducting fluid. It is assumed that the unsteady flow and heat transfer start at time $t = 0$. Keeping the origin fixed, the surface is stretched with velocity $U_w(x, t)$ along the x -axis. The stretching velocity $U_w(x, t)$ and the surface temperature $T_w(x, t)$ are given by

$$U_w(x, t) = ax/(1 - ct) \quad \text{and} \quad T_w(x, t) = T_\infty + bx/(1 - ct), \quad (1)$$

respectively, where a , b and c are constants with dimension time⁻¹ [16].

The governing equations for the problem can be written as [23]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u, \quad (3)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \\ &+ \frac{Q}{\rho c_p} (T - T_\infty), \end{aligned} \quad (4)$$

subject to the boundary conditions

$$\begin{aligned} U &= U_w, \quad V = 0, \quad T = T_w, \quad \text{at } y = 0, \\ U &\rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \quad (5)$$

where u and v are velocity components along the x -axis and y -axis, respectively. T is the fluid temperature in the boundary layer, t is time, v is the kinematic viscosity, ρ is the fluid density, α is the thermal diffusivity, c_p is the specific heat at constant pressure, μ is the coefficient of viscosity, Q is the volumetric heat generation/absorption rate and q_r is the radiative heat flux. To obtain similarity solution for Eqs. (2)–(5), the variable magnetic field and heat generation/absorption are assumed to be $B = B_0/\sqrt{1 - ct}$ and $Q = Q_0/(1 - ct)$ respectively, where B_0 and Q_0 are constants.

The radiative heat flux q_r , under Rosseland approximation [24], has the form

$$q_r = -(4\sigma^*/3k^*) \frac{\partial T^4}{\partial y} \quad (6)$$

where σ^* is the Stefan Boltzmann constant and k^* is the absorption coefficient. Temperature differences in the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding T^4 about T_∞ in Taylor's series and neglecting higher orders yield $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$

Substituting (6) and (7) in Eq. (4) gives

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha(1 + R) \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \\ &+ \frac{Q}{\rho c_p} (T - T_\infty), \end{aligned} \quad (8)$$

where $R = 16\sigma^*T_\infty^3/3k^*k$ is the radiation parameter and k is thermal conductivity.

The mathematical problem is simplified by introducing the following dimensionless functions f and θ , and the similarity variable η :

$$\begin{aligned} \eta &= y \sqrt{\frac{U_w}{vx}}, \quad \psi = \sqrt{U_w vx} f(\eta), \quad \theta(\eta) \\ &= (T - T_\infty)/(T_w - T_\infty) \end{aligned} \quad (9)$$

The equation of continuity is satisfied for the stream function $\psi(x, y)$ with the relations

$$u = \frac{\partial \psi}{\partial y} = (ax/1 - ct)f'(\eta), \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{va/1 - ct}f(\eta) \quad (10)$$

The mathematical problem defined by (2), (3), (8) and boundary conditions (5) are then transformed into a set of ordinary differential equations as follows:

$$f''' + ff'' - f'^2 - Mf' - A\left(f' + \frac{1}{2}\eta f''\right) = 0, \quad (11)$$

$$(1 + R)\theta'' + \text{Pr}(f\theta' - f'\theta) - \text{Pr}A\left(\theta + \frac{1}{2}\eta\theta'\right) + \text{Pr}(Ec(f'')^2 + \gamma\theta) = 0 \quad (12)$$

and corresponding boundary conditions are

$$f(0) = 0, f'(0) = 1, \theta(0) = 1 \quad \text{at } \eta = 0, \\ f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \quad (13)$$

where primes denote differentiation with respect to η . $A = c/a$ (Unsteady parameter), $M = \sigma B_0^2/\rho a$ (Magnetic parameter), $Ec = U_w^2/c_p(T_w - T_\infty)$ (Eckert Number), $\gamma = Q_0/\rho c_p a$ (Heat source parameter) and $Pr = \nu/\alpha$ (Prandtl number).

The physical quantities of interest are the skin friction coefficient

$$C_f = 2\tau_w/\rho U_w^2 \quad (14)$$

and the local Nusselt number

$$Nu_x = xq_w/k(T_w - T_\infty) \quad (15)$$

where the surface shear stress τ_w and the surface heat flux q_w are defined by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (16)$$

with μ being the dynamic viscosity. Using non-dimensional variables (9) in (16) we obtain

$$\frac{1}{2}C_f\sqrt{Re_x} = f''(0) \quad \text{and} \quad Nu_x/\sqrt{Re_x} \quad (17)$$

where $Re_x = U_w x/\nu$ is the local Reynolds number.

3. Numerical solution

Since Eqs. (11) and (12) are highly nonlinear, it is difficult to find the closed form solutions. Thus, the solutions of these equations with the boundary conditions (13) are solved numerically using the Keller box method. The convergence of the method depends on the choice of the initial guesses. The following initial guesses are chosen:

$$f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta}.$$

The choices of the initial guesses depend on the convergence criteria and the boundary conditions (13). The step size 0.01 is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process numerical computation, the Skin friction coefficient and Nusselt number which are respectively proportional to $f''(0)$ and $-\theta'(0)$ are presented in tabular form.

4. Results and discussion

In the numerical solutions, effects of Radiation and viscous dissipation on unsteady MHD boundary layer flow and heat transfer over a stretching sheet were considered. The momentum Eq. (11) coupled with energy Eq. (12) constitutes a set of non-linear ordinary differential equations for which obtaining closed form solution is difficult. Hence, Keller Box method [25–27] is used to solve this system subject to the boundary conditions (13). Velocity and temperature profiles were obtained and we applied the results to compute the Skin friction coefficient and Nusselt number in Eq. (17). The numerical results were discussed for the values of the parameters graphically and in tabular form. To verify the validity and accuracy of the result obtained, numerical results for the values of heat transfer rate at the surface are compared with Ishak [22], and Yusof et al. [23] for the case $R = 0$ and steady state case ($A = 0$). The numerical values (as shown in Table 1) are in excellent agreement with the values of Ishak [22] and Yusof et al. [23]. As it is shown in Table 1 the heat transfer coefficient increases with an increase of Prandtl number. This is true because by definition, Prandtl number is the ratio of kinematic viscosity to thermal diffusivity. An increase in the values of Prandtl number implies that momentum diffusivity dominates thermal diffusivity. Hence, the rate of heat transfer at the surface increases with increasing values of Prandtl number.

Effects of the Radiation Parameter (R), Prandtl number (Pr) on Skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ are shown in Table 2. It is noticed that as the radiation parameter R increases the magnitude of the Nusselt number $-\theta'(0)$ decreases. It is observed that as the Prandtl number Pr increases the magnitude of the Nusselt number $-\theta'(0)$

Table 1 Values of $-\theta'(0)$ for various values of A , M , R and Pr .

A	M	R	Pr	Ishak [22] Keeler box method	Yusof et al. [23] (Runge–Kutta–Fehlberg with shooting technique)	Present
0	0	0	0.72	0.8086	0.808631	0.8086308
			1	1.0000	1.000000	1.0000000
			3	1.9237	1.923683	1.9237161
			6.7	3.0003	3.000272	3.0003220
0	1	0	0.7	0.6897	0.689712	0.6897110
			1	0.8921	0.892147	0.8921452
			10	3.6170	3.616992	3.6170717
			0.7	1.0834	1.083386	1.0832785
1	0	0	7	3.7682	3.768235	3.7645541
			0.7	1.0500	1.049986	1.0499175
1	1	0	7	3.7164	3.716467	3.7136611
			0.7	–	0.708645	0.7086420
1	1	1	1	–	0.867918	0.8678333
			3	–	1.608920	1.6081314
			7	–	2.561119	2.5596874
			0.7	–	–	–

Table 2 Numerical values of the Skin Friction coefficient and Nusselt number for various values of A , M , R , Pr , Ec , γ .

A	M	R	Pr	Ec	γ	$-\theta'(0)$	$f''(0)$		
0.5	0.5	0.5	0.72	0.1	0.1	0.662541	-1.365633		
						0.5	-1.365633		
						1.0	-1.365633		
						0.2	-1.365633		
							0.3	0.552430	-1.365633
					1.0	0.1	0.1	0.807784	-1.365633
					3.0			1.532200	-1.365633
					5.0			2.035281	-1.365633
					7.0			2.442865	-1.365633
					10			2.954325	-1.365633
				1.0	0.72	0.1	0.1	0.636583	-1.538442
				1.5				0.615585	-1.693472
			1.0	0.5				0.800362	-1.497214
							0.911575	-1.619752	
1.5					0.556132	-1.365633			
0.5		1.0							
		1.5			0.485385	-1.365633			

increases. But Skin Friction Coefficient remains constant with increasing values of Radiation Parameter and Prandtl Number.

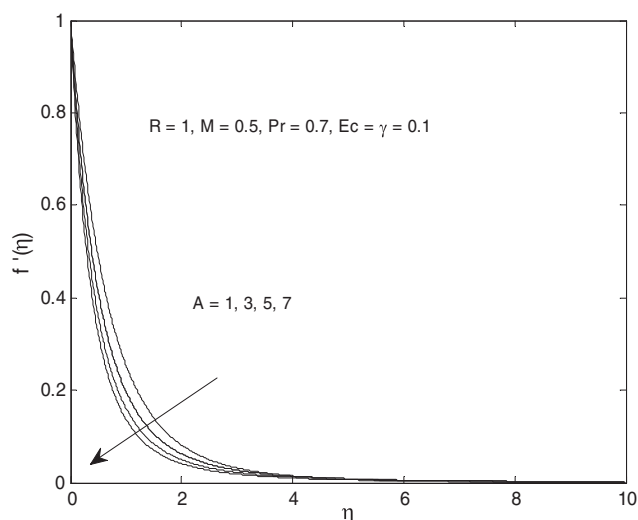
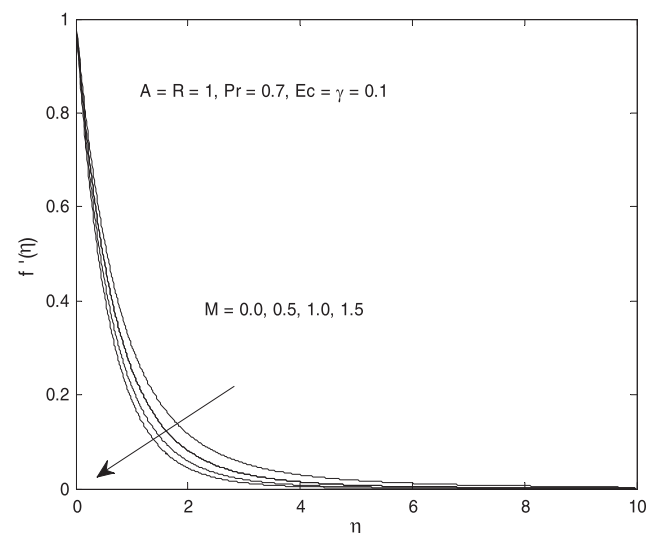
Effects of M and A on the velocity profiles are shown in Figs. 1 and 2. As expected, the velocity profiles increase with the increase in M . When M increases, it will also increase the Lorentz force which opposes the flow and leads to enhance deceleration of the velocity profiles. The momentum boundary layer thickness decreases with M or A , hence, induces an increase in the absolute value of the velocity gradient at the surface. Thus, the rate of heat transfer at the surface decreases with the presence of magnetic parameter and unsteadiness parameter.

Figs. 3 and 4 depict temperature profiles for various values of Prandtl number Pr and Unsteadiness parameter A . The Prandtl number Pr defines the ratio of momentum diffusivity to thermal diffusivity. It is noticed that an increase in Pr results a decrease of the thermal boundary layer thickness and in general lowers average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to

increasing the thermal conductivities, and heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr [29]. Hence in the case of smaller Prandtl numbers as the boundary layers are thicker the rate of heat transfer is reduced. Both figures show that the thermal boundary layer thickness decreases as Pr or A increases with increasing temperature gradient at the surface. Thus, the heat transfer rate at the surface increases with increasing values of Pr or A .

The temperature profile for various values of magnetic parameter M , radiation parameter R , Eckert number Ec , heat source parameter γ is presented in Figs. 5–8 respectively. Figs. 5 and 6 show effects of magnetic parameter M and radiation parameter R on the temperature θ profiles respectively. From these figures it can be seen that the absolute value of the temperature gradient at the surface decreases with an increase in M or R . So, the heat transfer rate at the surface decreases as M or R increases. As R increases the temperature profile also increases. The radiation parameter R is responsible to the thickening of the thermal boundary. This enables the fluid to release the heat energy from the flow region and causes the system to cool. This is true because the Rosseland approximation results in an increase in temperature. Fig. 7 shows the effect of viscous dissipation parameter Ec on temperature profile. The Eckert number Ec expresses the relationship between the kinetic energy in the flow and the enthalpy [28]. It embodies the conversion of Kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature profile. As Ec increases, the temperature profile also increases; from Fig. 7, we notice that an increase in the Eckert number Ec is to increase the temperature distribution. This is in conformity to the fact that the energy is stored in the fluid region as a consequence of dissipation due to viscosity and elastic deformation. The Temperature profiles for various values of heat source parameter γ are presented in Fig. 8. As γ increases, the temperature profiles also increase. It can be seen that the absolute value of temperature gradient at the surface decreases with an increase in M , R , Ec and heat source parameter γ .

Effects of Eckert Number Ec , M and Heat Source Parameter γ , M on Nusselt Number and Skin Friction

**Figure 1** Velocity profiles for different values of A .**Figure 2** Velocity profiles for different values of M .

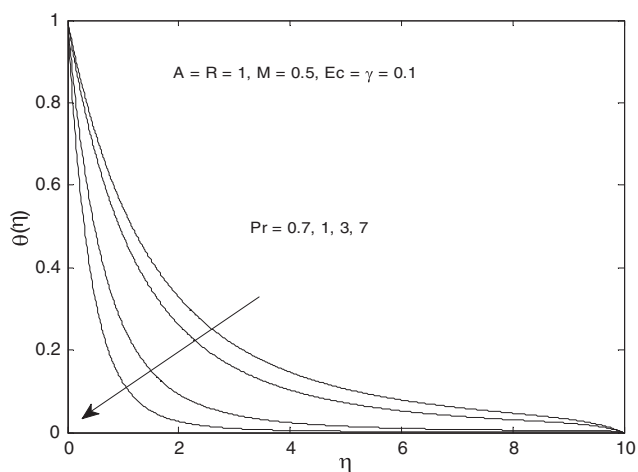


Figure 3 Temperature profiles for different values of Pr .

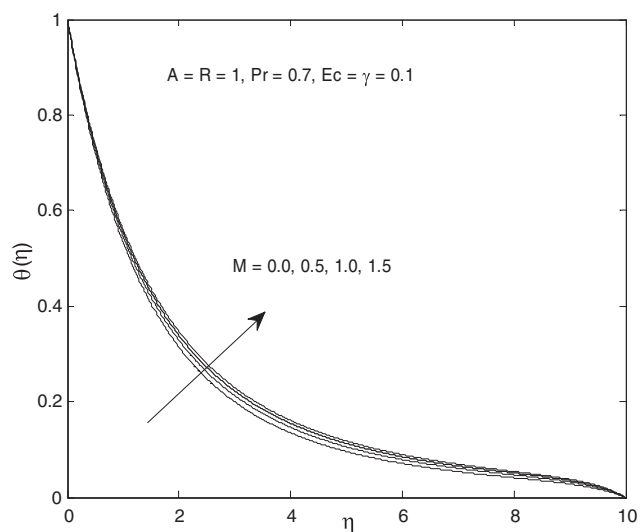


Figure 6 Temperature profiles for different values of R .

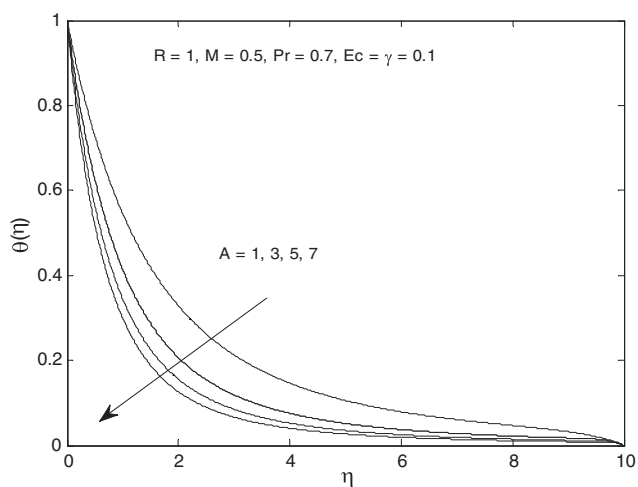


Figure 4 Temperature profiles for different values of A .

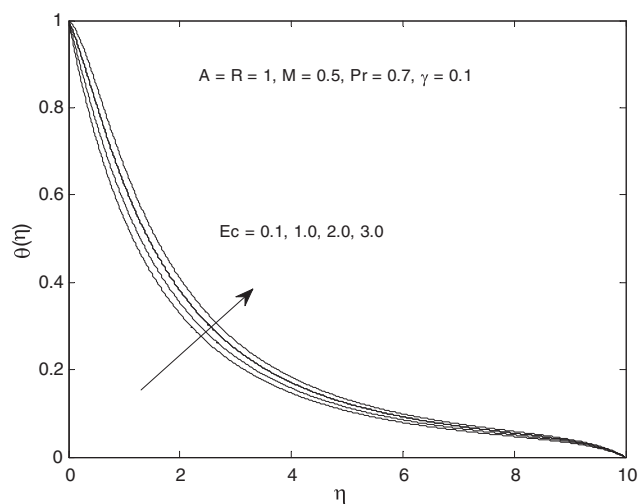


Figure 7 Temperature profiles for different values of Ec .

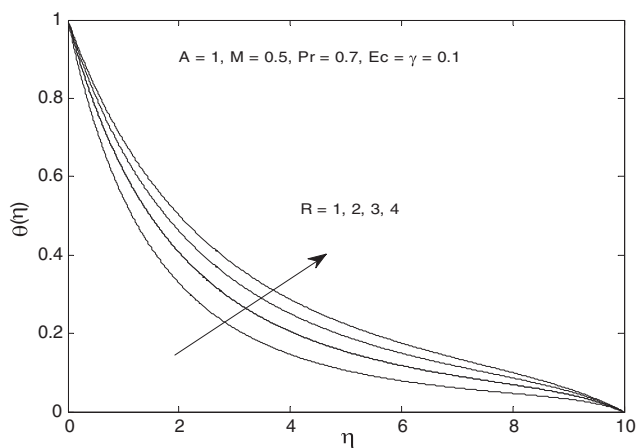


Figure 5 Temperature profiles for different values of M .

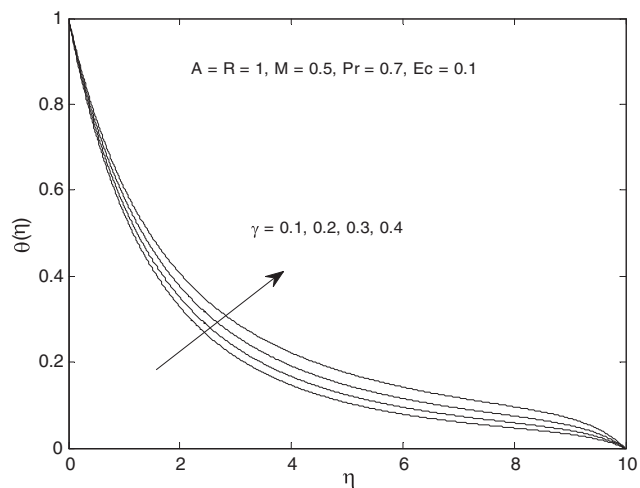


Figure 8 Temperature profiles for different values of γ .

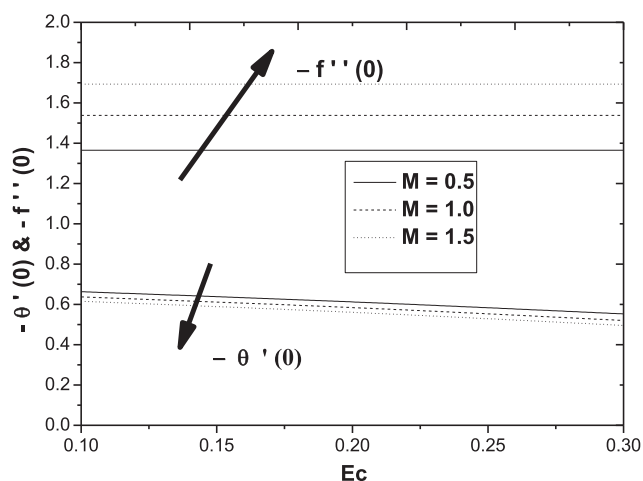


Figure 9 Effect of Ec and M on Nusselt Number and Skin Friction Coefficient.

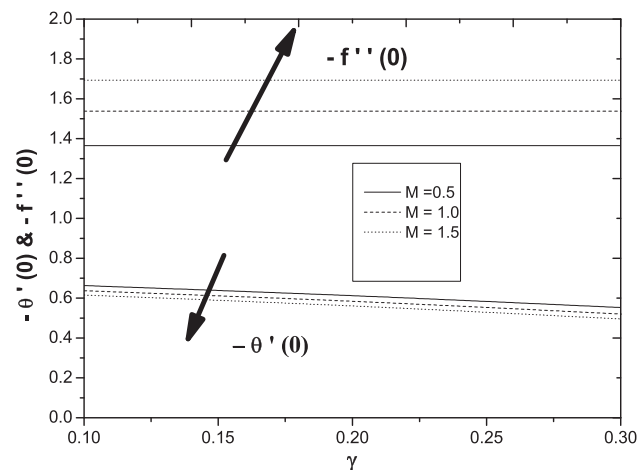


Figure 10 Effect of γ and M on Nusselt Number and Skin Friction Coefficient.

Coefficient are shown in Figs. 9 and 10 respectively. It is noticed that as the Eckert number Ec and M increase the magnitude of the Nusselt number $-\theta'(0)$ decreases whereas Skin Friction Coefficient increases. It is found that as the heat source parameter γ and M increase the magnitude of the Nusselt number $-\theta'(0)$ decreases whereas Skin Friction Coefficient increases.

5. Conclusion

The influence of thermal radiation and viscous dissipation on unsteady MHD fluid flow past a stretching sheet has been studied in this paper. Effects of radiation parameter, magnetic parameter, Eckert number, Prandtl number and heat source parameters on velocity profile, temperature profile, skin friction coefficient and the rate of wall heat transfer characteristics were observed. The skin friction coefficient decreases as A and M increase. And Nusselt number increases with Pr and A and decreases with Ec , M , R , γ . From our numerical results, the following conclusions may be drawn:

1. Velocity profiles decrease with increase in magnetic parameter M or unsteadiness parameter A .
2. Temperature distribution increases as the radiation parameter R or Eckert number Ec or heat source parameter γ increases.
3. The heat transfer rate at the surface increases with increasing values of unsteadiness parameter and Prandtl number.
4. The magnitude of the skin friction coefficient $f''(0)$ decreases with an increase in unsteady parameter A whereas the magnitude of the Nusselt number $-\theta'(0)$ increases.
5. The heat transfer rate at the surface decreases with increase in radiation parameter, magnetic parameter, Eckert number and heat source parameter.

References

- [1] Altan T, Oh S, Gegel H. Metal forming fundamentals and applications. Metals Park, OH: American Society of Metals; 1979.
- [2] Fisher EG. Extrusion of plastics. New York: Wiley; 1976.
- [3] Tidmore Z, Klein I. Engineering principles of plasticating extrusion, polymer science and engineering series. New York: Van Nostrand; 1970.
- [4] Sakiadis BC. Boundary layer behaviour on continuous solid surface I, boundary-layer equations for two dimensional and axisymmetric flow. *AIChE J* 1961;7:26–8.
- [5] Sakiadis BC. Boundary layer behaviour on continuous solid surface II, boundary layer behaviour on continuous flat surface. *AIChE J* 1961;7:221–35.
- [6] Salleh MZ, Nazar R, Pop I. Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating. *J Taiwan Inst Chem Eng* 2010;41:651–5.
- [7] Abbas Z, Hayat T. Radiation effects on MHD Flow in a porous space. *Int J Heat Mass Trans* 2008;51:1024–33.
- [8] Ghaly AY. Radiation effects on a certain MHD free convection flow. *Chaos Solitons Fractals* 2002;13:1843–50.
- [9] Subhashini SV, Samuel N, Pop I. Effects of Buoyancy assisting and opposing flows on mixed convection boundary layer flow over a permeable vertical surface. *Int Commun Heat Mass Trans* 2011;38:499–503.
- [10] Raptis A. Effect of thermal radiation on MHD flow. *Appl Math Comput* 2004;153:645–9.
- [11] Chiam TC. Hydro magnetic flow over a surface stretching with a power-law velocity. *Int J Eng Sci* 1995;33(3):429–35.
- [12] Chen CH. On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation. *Int J Heat Mass Trans* 2010;53:4264–73.
- [13] Magyari E, Keller B. Exact solutions for self similar boundary layer flows induced by permeable stretching walls. *Eur. J. Mech. B-Fluids* 2000;19:109–22.
- [14] Fang T, Zhang J, Zhong Y. Boundary layer flow over a stretching sheet with variable thickness. *Appl Math Comput* 2012;218:7241–52.
- [15] Liao S. A new branch of solutions of boundary layer flows over an impermeable stretched plate. *Int J Heat Mass Trans* 2010;48:2529–39.
- [16] Anderson HI, Aarseth JB, Dandapat BS. Heat transfer in a liquid film on an unsteady stretching surface. *Int J Heat Mass Trans* 2000;43:69–74.
- [17] Elbashbeshy EMA, Bazid MAA. Heat transfer over an unsteady stretching surface. *Heat Mass Trans* 2004;41:1–4.
- [18] Ishak, Nazar R, Pop I. Heat transfer over an unsteady stretching surface with prescribed heat flux. *Can. J. Phys.* 2008;86:853–5.
- [19] Ishak, Nazar R, Pop I. Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature. *Non-Linear Anal: Real World Appl* 2009;10:2909–13.

- [20] Ali A, Mehmood A. Homotopy analysis of unsteady boundary layer flow adjacent to permeable stretching surface in a porous medium. *Commun Nonlinear Sci Numer Simul* 2008;13:340–9.
- [21] Ganeswara M. Reddy, unsteady radiative convective boundary layer flow of a casson fluid with variable thermal conductivity. *J Eng Phys Thermophys* 2015;88(1).
- [22] Ishak. Unsteady MHD flow and heat transfer over a stretching plate. *J Appl Sci* 2010;10:2127–31.
- [23] Yusof Zanariah Mohd, Soid Siti Khuzaimah, Aziz Ahmad Sukri Abd, Kechil Seripah Awang. Radiation effect on unsteady MHD flow over a stretching surface. *World Acad Sci, Eng Technol* 2012;6:12–27.
- [24] Ei-Aziz MA. Radiation effect on the flow and heat transfer over an unsteady stretching sheet. *Int Commun Heat Mass Trans* 2009;36:521–4.
- [25] Cebeci T, Bradshaw P. Physical and computational aspects of convective heat transfer. New York: Springer Verlag; 1988.
- [26] Keller HB. Numerical methods for two point boundary value problems. New York: Dover publications; 1992.
- [27] Keller HB. Accurate difference methods for non-linear two point boundary value problems. *SIAM J Num Anal* 1974;11:305–20.
- [28] Ganeswara Reddy M. Influence of thermal radiation, viscous dissipation and hall current on MHD convection flow over a stretched vertical flat plate. *Ain Shams Eng J* 2014;5:169–75.
- [29] Haile E, Shankar B. Heat and mass transfer through a porous media of MHD Flow of nanofluids with thermal radiation, viscous dissipation and chemical reaction effects. *Am Chem Sci J* 2014;4:828–46.



Ganeswara Reddy Machireddy was born and brought up in the district of Chittoor, Andhra Pradesh, India. He obtained the B.Sc. Computers and M.Sc. degrees in Mathematics from Sri Venkateswara University. He was awarded PhD degree in Fluid Mechanics by Sri Venkateswara University in 2009. He is serving the Department of Mathematics, Acharya Nagarjuna University, Andhra Pradesh as Assistant Professor since 2009.

Besides teaching he is actively engaged in

research in the field of Fluid mechanics particularly, in Heat transfer, boundary layer flows, heat and mass transfer in porous/non-porous media. His research interest also covers the Elliptic curve cryptography.



Padma Polarapu was born in Khammam district, Andhra Pradesh. She obtained B.Sc. (Mathematics, Physics and Computer Science) from Kakatiya University in the year 1992. She got University Second Rank in B.Sc. She completed M.Sc. (Applicable Mathematics) at Sri Padmavathi Mahila Viswa Vidhyalayam, Tirupati in 1994. She got gold medal in P.G. from Sri Padmavathi Mahila Viswa Vidhyalayam, Tirupati. She obtained M.Phil from Madurai Kamaraj University, Madurai.

She completed Bachelor of Education degree at Annamalai University, Tamilnadu. She is pursuing PhD under the Guidance of Dr. M. Ganeswara Reddy Acharya Nagarjuna University Campus, Ongole, Andhra Pradesh. She is working as Lecturer in Mathematics at S.R. Government A & S College, Kothagudem, Khammam, India.



Bandari Shankar is presently working as Professor of Mathematics, Fluid Dynamics specialised at Osmania University, Hyderabad, India. Dr. Bandari Shankar obtained his PhD degree in Mathematics in the area of Computational Fluid Dynamics (CFD) from Osmania University, Hyderabad, India in 1991. He also did his Bachelor's and Master's Degree in Mathematics from Osmania University Hyderabad, India. His research interests are Fluid Dynamics, Porous

Media, Magnetohydrodynamics, Heat and Mass Transfer, Computational Fluid Dynamics. Prof. Bandari Shankar has published 35 refereed research papers so far in various national, international journals and conferences. He has produced 15 PhD's so far under his supervision.